CLAIMS:

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1. A computer implemented method for relating a price or value of a plurality of securities associated with an underlying asset, the rate of return on said securities and the risk attributes of said securities, the method comprising the steps of:

determining a risk premium incorporated in the rate of return for each security;

designating that a priced risk factor incorporated in the risk premium for each security is the volatility of returns, measured over discrete time, and that the price per unit of this risk factor is the same for two or more of the said securities; and

defining a model comprising data representing relationships between the risk premiums determined for each security.

- 2. The computer implemented method of claim 1, wherein at least one of said plurality of securities is a debt-type instrument, and further comprising analysing a yield spread associated with the debt-type instrument and identifying a default loss component and a risk premium component of said yield spread.
- 3. The computer implemented method of claim 1, further comprising fitting the model.
- 4. The computer implemented method of claim 3, further comprising providing as output to a user parameters of the fitted model.
- 5. The computer implemented method of claim 1, wherein the rate of return for a security (or securities) issued by, or referenced to, a firm is analysed utilising an estimate of the expected default loss of another, debt-type security (security j) issued by, or referenced to, the firm, the method further comprising the steps of:

determining the rate of return on security j (r_j) by reference to the promised yield on said security (y_i) and the expected default loss (EDL_j) on said security where:

$$r_i = y_j - EDL_j$$

calculating the excess return for said security j as equal to $r_j - r$, where r is the risk free rate of return;

calculating the exposure of each security to each priced risk factor (m);

calculating a price per unit of risk (λ_m) for each priced risk factor (m) in which each λ_m is the same for two or more securities issued by, or referenced to, the firm and such that the product of the risk exposures for security j and the prices per unit of risk equals the excess return for security j, and similarly for any other security for which an estimate of the excess return is available;

designating that one of the priced risk factors relates to the volatility of the rate of return on securities estimated over a discrete time period and is specific to securities issued by, or referenced to, the firm;

calculating the excess rate of return for all of the other securities being analysed, other than j, based at least partly on their exposure to each priced risk factor and the price per unit of risk for each risk factor;

fitting the model; and

providing as output to a user parameters of interest from the fitted model.

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6. The computer implemented method of claim 5, wherein the only priced risk factor comprises the volatility of returns and is implemented by:

designating the relationship between the firm specific price of volatility risk (λ_{σ}) , the rate of return for $j(r_j)$, the volatility of returns for $j(\sigma_j)$ and the risk free rate of return (r) as:

$$\lambda_{\sigma} = \frac{r_{j} - r}{\sigma_{j}}$$

designating the rate of return (r_k) on another class, or classes, of security (k) issued by, or referenced to, the firm as:

$$r_k = r + \lambda_{\sigma} \sigma_k$$

designating, where security class or classes k are debt-type securities, the default loss on said securities by combining the promised yield on said securities (y_k) and their rate of return (r_k) as follows:

$$EDL_k = y_k - r_k$$

fitting the model; and

providing as output to a user parameters of interest from the fitted model.

7. A computer implemented method of estimating the covariance of returns for two assets and using said covariance output as a measure of credit risk of one of the assets.

- 8. The computer implemented method of claim 7, wherein the two assets are securities issued by, or referenced to, the same firm and using said covariance output as a measure of credit risk of the security that ranks highest in priority upon a liquidation or default event.
- 9. A computer implemented method of estimating the expected default loss of a security and using said estimate of expected default loss as a measure of the covariance of returns for that security and another security issued by, or referenced to, the same firm wherein the first security ranks higher in priority upon a liquidation or default event.
- 15 10. The computer implemented method of any one of claims 7 to 9, wherein the annualised expected default loss (EDL_j) on one of the said securities, security j, is designated as:

$$EDL_{j} = \ln \left(\rho_{jk} \sqrt{\left(e^{\sigma_{j}^{2}T} - 1\right)\left(e^{\sigma_{k}^{2}T} - 1\right)} + 1 \right) / T$$

20 where:

- j is the class or classes of the firm's debt-type or similar securities issued by, or referenced to, the firm for which the expected default loss is being estimated
- k is the class or classes of security issued by, or referenced to, the firm that rank behind security j in terms of priority upon a liquidation or default event
- T is the time horizon of interest to the user, in years
 - σ_j is the standard deviation of rates of return, per annum, of j
 - σ_k is the standard deviation of rates of return, per annum, of k
 - ρ_{jk} is the correlation coefficient of the rates of return for j and k;

the model is fitted; and parameters of interest from the fitted model are output to a user.

11. The computer implemented method of any one of claims 7 to 9, wherein the annualised expected default loss (EDL_j) on one of the said securities, security j, is designated as:

$$EDL_{j} = \rho_{jk}\sigma_{j}\sigma_{k}$$

the model is fitted; and

- parameters of interest from the fitted model are output to a user.
 - 12. The computer implemented method of claim 7, wherein the two assets are portfolios or indices in respect of different types of security and using said covariance output as a measure of credit risk.

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13. A computer implemented method for estimating the correlation of returns for two securities issued by, or referenced to, a firm by relating the said correlation to computer generated estimates of the variance of the said securities and the expected default loss of one of the said securities.

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14. The computer implemented method of claim 13, wherein the correlation (ρ_{jk}) of the returns for the two said securities, j and k, is designated as:

$$\rho_{jk} = EDL_jT / \sqrt{\left(e^{\sigma_j^2T} - 1\right)\left(e^{\sigma_k^2T} - 1\right)}$$

where:

- j is the class or classes of the firm's debt or similar securities issued by, or referenced to, the firm for which the expected default loss is being estimated
- k is the class or classes of security issued by, or referenced to, the firm that rank behind security j in terms of priority upon a liquidation or default event
- T is the time horizon of interest to the user, in years

 σ_i is the standard deviation of rates of return, per annum, of j

 σ_k is the standard deviation of rates of return, per annum, of k

 EDL_i the annualised expected default loss on security j;

the model is fitted; and

5 parameters of interest from the fitted model are output to a user.

15. The computer implemented method of claim 13, wherein the correlation (ρ_{jk}) of the returns for two said securities j and k, is designated as:

$$\rho_{jk} = EDL_j / \sigma_j \sigma_k ;$$

the model is fitted; and

parameters of interest from the fitted model are output to the user.

16. The computer implemented method of claim 1, wherein one or more of the securities is an option, the method further comprising the steps of:

specifying the real world distribution process that the returns on the underlying asset are expected to follow;

calculating the expected real world probability of the option being exercised;

calculating the expected mean, standard deviation and other higher moments of interest of the option, at the time the option is expected to be exercised;

using the aforesaid parameters to calculate the expected real world pay off from the option;

discounting back to present value (as at the chosen evaluation date) the pay off from the option using a risk adjusted discount rate, where said risk adjusted discount rate includes a risk premium for the expected standard deviation (measured over discrete time) of the expected option pay off, for such other higher moments of interest to the user and adjustments for any other factors specified by a user, such that the price per unit of risk, for each risk factor, is equated for all of the options being evaluated, for the underlying asset and for any other securities of interest referenced thereto;

providing as output to a user parameters of interest from the fitted model.

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17. The computer implemented method of claim 1, wherein one or more of the securities is an option, the method further comprising the steps of:

specifying the real world distribution process that the returns on the underlying asset, upon which the option is contingent, are expected to follow, using initial guesses for the values of the distribution parameters;

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calculating the expected real world probability of the option being exercised;

calculating the expected mean, standard deviation and other higher moments of interest of the option, at the time the option is expected to be exercised;

using the aforesaid parameters to calculate the expected real world pay off from each option;

discounting back to present value (as at the chosen evaluation date) the expected pay offs from each option using a risk adjusted discount rate specific to each option, where said risk adjusted discount rate includes a risk premium for the expected standard deviation of the expected option pay off, for such other higher moments of interest to the user and adjustments for any other factors specified by a user, such that the price per unit of risk, for each risk factor, is equated for all of the options being evaluated, for the underlying asset and for any other securities of interest referenced thereto;

iterating the above process, by varying the model parameter values, until the model is fitted; and

providing as output to a user parameters of interest from the fitted model.

- 18. The computer implemented method of claim 16 or claim 17, further comprising the step of using the estimated values for the rate of return, standard deviation, other higher moments of interest and any other factors specified by a user for the asset as input to price or value other options contingent on the same or similar assets.
- 19. The computer implemented method of claim 1, wherein a user applies an optiontheoretic model of the firm, the method further comprising the steps of:

determining a plurality of input parameters, the parameters including a risk premium in the rate of return for each security issued by, or referenced to, the firm;

defining relationships between said parameters;

fitting the model; and providing as output to a user parameters of interest from the fitted model.

20. The computer implemented method of claim 19, further comprising the steps of: specifying the real world distribution process that the returns on the firm's assets are expected to follow;

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specifying a default point representing the value of the firm's assets at which the firm is expected to default;

calculating the expected real world probability of the default point being met; calculating the expected mean, standard deviation and other higher moments of interest of the securities being analysed, having regard to the distribution process modelled for the firm's assets and the default point, at the time horizon of interest;

using the aforesaid parameters to calculate the expected real world pay off of the securities being analysed, at the time horizon of interest;

discounting back to present value (as at the chosen evaluation date) the expected pay offs of each security being analysed using a risk adjusted discount rate, where said risk adjusted discount rate includes a risk premium for the expected standard deviation of the expected pay off from the security, for such other higher moments of interest to the user and adjustments for any other factors specified by a user, such that the price per unit of risk, for each such risk factor, is equated for two or more securities issued by, or referenced to, the firm;

fitting the model; and providing as output to a user parameters of interest from the fitted model.

25 21. The computer implemented method of claim 19 or claim 20, further comprising the steps of:

defining additional multi-variate equations representing relationships between some or all of the variables used in the models of claim 19 or claim 20; and

solving all of the multi-variate equations and the said model to calculate the remaining unknown variables in the equations and the model.

22. The computer implemented method of claim 21, wherein at least one of the unknown inputs included in one or more additional multi-variate equations comprises or represents a statistical moment of one of the securities issued by, or referenced to the firm.

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23. The computer implemented method of claim 21, wherein at least one of the unknown inputs included in one or more additional multi-variate equations comprises or represents the correlation between the returns of a pair of securities issued by, or referenced to, the firm.

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24. The computer implemented method of claim 21, wherein at least one of the unknown inputs included in one or more additional multi-variate equations comprises or represents the covariance between the returns of a pair of securities issued by, or referenced to, the firm.

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25. The computer implemented method of claim 21, wherein at least one of the unknown inputs included in one or more additional multi-variate equations comprises or represents the correlation between the returns of a security issued by, or referenced to, the firm and the returns of the total firm.

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26. The computer implemented method of claim 21, wherein at least one of the unknown inputs included in one or more additional multi-variate equations comprises or represents the covariance between the returns of a security issued by, or referenced to, the firm and the returns of the total firm.

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27. The computer implemented method of claim 21, wherein at least one of the unknown inputs included in one or more additional multi-variate equations comprises or represents the expected probability of default.

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28. The computer implemented method of claim 21, wherein at least one of the unknown inputs included in one or more additional multi-variate equations comprises or

represents the expected loss given default on a debt-type security issued by, or referenced to, the firm.

- 29. The computer implemented method of claim 21, wherein at least one of the unknown inputs included in one or more additional multi-variate equations comprises or represents the expected default loss on a debt-type security issued by, or referenced to, the firm.
- 30. The computer implemented method of claim 19 or claim 20, further comprising the steps of generating one or more parameters from the model and solving the model so that the said parameters equal values specified by a user, where one of the said parameters is a statistical moment of the returns of one of the securities issued by, or referenced to, the firm.
- 15 31. The computer implemented method of claim 19 or claim 20, further comprising the steps of generating one or more parameters from the model and solving the model so that the said parameters equal values specified by a user, where one of the said parameters is the correlation between the returns of a pair of securities issued by, or referenced to, the firm.

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32. The computer implemented method of claim 19 or claim 20, further comprising the steps of generating one or more parameters from the model and solving the model so that the said parameters equal values specified by a user, where one of the said parameters is the covariance between the returns of a pair of securities issued by, or referenced to, the firm.

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33. The computer implemented method of claim 19 or claim 20, further comprising the steps of generating one or more parameters from the model and solving the model so that the said parameters equal values specified by a user, where one of the said parameters is the correlation between the returns of a security issued by, or referenced to, the firm and the returns of the total firm.

34. The computer implemented method of claim 19 or claim 20, further comprising the steps of generating one or more parameters from the model and solving the model so that the said parameters equal values specified by a user, where one of the said parameters is the covariance between the returns of a security issued by, or referenced to, the firm and the returns of the total firm.

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- 35. The computer implemented method of claim 19 or claim 20, further comprising the steps of generating one or more parameters from the model and solving the model so that the said parameters equal values specified by a user, where one of the said parameters is the expected probability of default.
- 36. The computer implemented method of claim 19 or claim 20, further comprising the steps of generating one or more parameters from the model and solving the model so that the said parameters equal values specified by a user, where one of the said parameters is the expected loss given default on a debt-type security issued by, or referenced to, the firm.
- 37. The computer implemented method of claim 19 or claim 20, further comprising the steps of generating one or more parameters from the model and solving the model so that the said parameters equal values specified by a user, where one of the said parameters is the expected default loss on a debt-type security issued by, or referenced to, the firm.
- 38. The computer implemented method of any one of claims 2, 5 or 6 or any one of claims 19 to 37, wherein the value for, or the additional multi-variate equation for solving for, the default loss on a security is derived from applying the computer implemented method of any one of claims 7, 8 or 10.
- 39. The computer implemented method of any one of claims 2, 5 or 6 or any one of claims 19 to 37, wherein the value for, or the additional multi-variate equation for solving for, the correlation between a pair of securities is derived from applying the computer implemented method of any one of claims 13 to 15.

40. The computer implemented method of any one of claims 19 to 37, wherein the values for, or the additional multi-variate equations for solving for, the rate of return, standard deviation, other higher moments of interest and any other relevant factors specified by a user for one or more of the securities are derived from applying the computer implemented method of claims 16 or claim 17.

41. The computer implemented method of any one of claims 16, 17 or 20, wherein the real world distribution process that the returns on the firm (or underlying asset) are expected to follow is modelled as a specified statistical distribution, wherein the mean, standard deviation and other higher moments of interest of the portions of that distribution relevant to a security are estimated using closed-form type formula solutions or numerical approximations appropriate for the specified statistical distribution process.

42. The computer implemented method of claim 41, wherein the real world statistical distribution process that the returns on the firm (or underlying asset) are expected to follow is the normal distribution.

43. The computer implemented method of claim 42, wherein the firm has, or is treated as having, only a single class of zero coupon debt on issue and the model is fitted such that:

$$\frac{\ln\left(\frac{V_{0}e^{\eta rT}[1-N(d_{1})]+B_{0}e^{\gamma rT}N(d_{2})}{B_{0}}\right)/T-r}{\sigma_{B}} = \frac{\ln\left(\frac{V_{0}e^{\eta rT}N(d_{1})-B_{0}e^{\gamma rT}N(d_{2})}{S_{0}}\right)/T-r}{\sigma_{S}}$$

where:

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- S_n is the value of the equity of the firm at time n
 - V_n is the value of the firm's assets at time n and the value of the firm's assets is the sum of the values of the firm's debt (B) and equity (S)
 - X is the face value of the firm's debt (B), which is assumed to be a single zerocoupon bond, at maturity
- T is the user selected time horizon, in years

- r_{ν} is the rate of return on the firm's assets, per annum
- y is the promised yield on the firm's debt, per annum

$$d_{1} = \left(\left[\ln \left(\frac{V_{0}}{X} \right) + r_{v} T \right] / \sigma_{v} \sqrt{T} \right) + (1/2) \left(\sigma_{v} \sqrt{T} \right)$$

$$d_2 = d_1 - \sigma_V \sqrt{T}$$

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- 5 N(.) is the cumulative probability of the standard normal distribution with d_1 or d_2 as the upper limit
 - r is the risk free rate of return, per annum
 - σ_{ν} is the standard deviation of rates of return on the firm's assets, per annum
 - $\sigma_{\scriptscriptstyle B}$ is the standard deviation of rates of return on the firm's debt, per annum
- 10 σ_s is the standard deviation of rates of return on the firm's equity, per annum.
 - 44. A computer implemented method for applying an option-theoretic model of a firm comprising the steps of generating one or more parameters from the model, estimated over a discrete time period, and solving the model so that the said parameters equal values specified by a user.
 - 45. The computer implemented method of claim 44, wherein one of the said parameters is a statistical moment of the returns of one or more of the securities issued by, or referenced to, the firm.
 - 46. The computer implemented method of claim 44, wherein one of the said parameters is the correlation between the returns of a pair of securities issued by, or referenced to, the firm.
- 25 47. The computer implemented method of claim 44, wherein one of the said parameters is the covariance between the returns of a pair of securities issued by, or referenced to, the firm.
- 48. The computer implemented method of claim 44, wherein one of the said parameters is the correlation between the returns of a security issued by, or referenced to, the firm and the returns of the total firm.

The computer implemented method of claim 44, wherein one of the said 49. parameters is the covariance between the returns of a security issued by, or referenced to, the firm and the returns of the total firm.

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The computer implemented method of any one of claims 19 to 22, claim 30 or 50. claims 41 to 49, wherein the formula for calculating additional parameters, being instantaneous volatility, for calibration with the model comprise:

$$\sigma_{\scriptscriptstyle B} = \sigma_{\scriptscriptstyle V} \, \frac{V_{\scriptscriptstyle 0}}{B_{\scriptscriptstyle 0}} \, e^{(r_{\scriptscriptstyle V} - r_{\scriptscriptstyle B})T} \left[1 - N(d_{\scriptscriptstyle 1}) \right]$$

$$\sigma_{S} = \sigma_{V} \frac{V_{0}}{S_{0}} e^{(r_{V} - r_{S})T} N(d_{1})$$

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The computer implemented method of any one of claims 19 to 22, claims 30 to 51. 34, or claims 41 to 49, wherein the formula for calculating additional parameters, being discrete time volatility, correlation and covariance, for calibration with the model comprise:

$$\sigma_{B} = \sqrt{\ln\left(\frac{V_{T}^{2} \left[1 - N(d_{3})\right] e^{\sigma_{V}^{2}T} + X^{2} N(d_{2})}{B_{T}^{2}}\right) / T}$$

$$\sigma_{S} = \sqrt{\ln\left(\frac{V_{T}^{2}N(d_{3})e^{\sigma_{V}^{2}T} - 2V_{T}XN(d_{1}) + X^{2}N(d_{2})}{S_{T}^{2}}\right)} / T$$

$$\rho_{BS} = \frac{X - B_{T}}{B_{T}\sqrt{\left(e^{\sigma_{S}^{2}T} - 1\right)\left(e^{\sigma_{B}^{2}T} - 1\right)}}$$

$$\rho_{VB} = \frac{V_{T}\left[1 - N(d_{3})\right]e^{\sigma_{V}^{2}T} + XN(d_{1}) - B_{T}}{B_{T}\sqrt{\left(e^{\sigma_{V}^{2}T} - 1\right)\left(e^{\sigma_{B}^{2}T} - 1\right)}}$$

$$\rho_{VS} = \frac{V_{T}N(d_{3})e^{\sigma_{V}^{2}T} - XN(d_{1}) - S_{T}}{S_{T}\sqrt{\left(e^{\sigma_{V}^{2}T} - 1\right)\left(e^{\sigma_{S}^{2}T} - 1\right)}}$$

$$\sigma_{BS} = \rho_{BS}\sigma_{B}\sigma_{S}$$

$$\sigma_{VB} = \rho_{VB}\sigma_{V}\sigma_{B}$$

$$\sigma_{VS} = \rho_{VS}\sigma_{V}\sigma_{S}$$

where the additional term is:

$$d_3 = d_1 + \sigma_V \sqrt{T}$$

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- 52. A system for relating a price or value of a plurality of securities associated with an underlying asset, the rate of return on said securities and the risk attributes of said securities, the system comprising:
- a computer-readable memory;
 - a risk analysis unit operative to designate that a priced risk factor incorporated in the risk premium for each security is the volatility of returns, measured over discrete time;

a risk pricing unit operative to:

- determine a risk premium incorporated in the rate of return for each security; and
 - designate that the price per unit of this risk factor is the same for two or more of the said securities;
- a financial modelling unit operative to define a model comprising data 25 representing relationships between the risk premiums determined for each security and

store the model in the computer-readable memory; and a user interface device operative to exchange information with a user.

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- 53. The computer system of claim 52, wherein at least one of said plurality of securities is a debt-type instrument, and the risk pricing unit is further operative to analyse a yield spread associated with the debt-type instrument and identify a default loss component and a risk premium component of said yield spread.
- 54. The computer system of claim 52, wherein the financial modelling unit is further operative to fit the model.
 - 55. The computer system of claim 54, wherein the user interface is further operative to provide as output to a user parameters of the fitted model.
- 15 56. The computer system of claim 52, wherein an estimate of the expected default loss of another, debt-type security (security *j*) issued by, or referenced to, the firm, is utilised in analysing the rate of return for a security (or securities) issued by, or referenced to, a firm;

the financial modelling unit is further operative to determine the rate of return on security $j(r_j)$ by reference to the promised yield on said security (y_j) and the expected default loss (EDL_j) on said security where $r_j = y_j - EDL_j$;

the risk pricing unit is further operative to calculate the excess return for said security j as equal to $r_j - r$, where r is the risk free rate of return;

the risk analysis unit is further operative to calculate the exposure of each security to each priced risk factor (m);

the risk pricing unit is further operative to calculate a price per unit of risk (λ_m) for each priced risk factor (m) in which each λ_m is the same for two or more securities issued by, or referenced to, the firm and such that the product of the risk exposures for security j and the prices per unit of risk equals the excess return for security j, and similarly for any other security for which an estimate of the excess return is available;

the risk analysis unit is further operative to designate that one of the priced risk factors relates to the volatility of the rate of return on securities estimated over a discrete

time period and is specific to securities issued by, or referenced to, the firm;

the risk pricing unit is further operative to calculate the excess rate of return for all of the other securities being analysed, other than *j*, based at least partly on their exposure to each priced risk factor and the price per unit of risk for each risk factor;

the financial modelling unit is further operative to fit the model; and

the user interface is further operative to provide as output to a user parameters of interest from the fitted model.

57. The computer system of claim 56, wherein the only risk factor priced in the said system comprises the volatility of returns;

the risk pricing unit is further operative to designate the relationship between the firm specific price of volatility risk (λ_{σ}) , the rate of return for j (r_j) , the volatility of returns for j (σ_i) and the risk free rate of return (r) as:

$$\lambda_{\sigma} = \frac{r_{j} - r}{\sigma_{j}};$$

the risk pricing unit is further operative to designate the rate of return (r_k) on another class, or classes, of security (k) issued by, or referenced to, the firm as:

$$r_k = r + \lambda_\sigma \sigma_k$$
;

the risk analysis unit is further operative to designate, where security class or classes k are debt-type securities, the default loss on said securities by combining the promised yield on said securities (y_k) and their rate of return (r_k) as follows:

$$EDL_k = y_k - r_k;$$

the financial modelling unit is further operative to fit the model; and the user interface is further operative to provide as output to a user parameters of interest from the fitted model.

58. A system for measuring credit risk, the system comprising:

a computer-readable memory; and

a processing unit operative to estimate the covariance of returns for two assets, wherein said covariance is used as a measure of credit risk of one of the assets.

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59. The computer system of claim 58, wherein the processing unit is further operative to analyse, as the said two assets, two securities issued by, or referenced to, the same firm, wherein said covariance output is used as a measure of credit risk of the security that ranks highest in priority upon a liquidation or default event.

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60. A system for estimating the covariance of securities, the system comprising: a computer-readable memory;

a processing unit operative to estimate the expected default loss of a security, wherein said estimate of expected default loss is used as a measure of the covariance of returns for that security and another security issued by, or referenced to, the same firm wherein the first security ranks higher in priority upon a liquidation or default event.

61. The computer system of any one of claims 58 to 60, wherein: the financial modelling unit is further operative to designate the annualised expected default loss (*EDL_j*) on one of the said securities, security *j*, as:

$$EDL_{j} = \ln \left(\rho_{jk} \sqrt{\left(e^{\sigma_{j}^{2}T} - 1\right)\left(e^{\sigma_{k}^{2}T} - 1\right)} + 1 \right) / T$$

where:

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- j is the class or classes of the firm's debt-type or similar securities issued by, or referenced to, the firm for which the expected default loss is being estimated
- k is the class or classes of security issued by, or referenced to, the firm that rank behind security j in terms of priority upon a liquidation or default event

- T is the time horizon of interest to the user, in years
- σ_j is the standard deviation of rates of return, per annum, of j
- $\boldsymbol{\sigma}_k$ is the standard deviation of rates of return, per annum, of k
- ρ_{jk} is the correlation coefficient of the rates of return for j and k; the financial modelling unit is further operative to fit the model; and

the user interface is further operative to output parameters of interest from the fitted model to a user.

62. The computer system of any one of claims 58 to 60, wherein the financial modelling unit is further operative to designate the annualised expected default loss (EDL_i) on one of the said securities, security j, as:

$$EDL_{j} = \rho_{jk}\sigma_{j}\sigma_{k}$$

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the financial modelling unit is further operative to fit the model; and

- the user interface is further operative to output parameters of interest from the fitted model to a user.
 - 63. The computer system of claim 58, wherein the processing unit is further operative to analyse, as the two assets, portfolios or indices in respect of different types of security, wherein said covariance is used as a measure of credit risk.
 - A system for estimating the correlation of securities, the system comprising: a computer-readable memory;
 - a processing unit operative to estimate the correlation of returns for two securities issued by, or referenced to, a firm by relating the said correlation to computer generated estimates of the variance of the said securities and the expected default loss of one of the said securities.
- 65. The computer system of claim 64, wherein the processing unit is further operative to:

designate the correlation (ρ_{jk}) of the returns for the two said securities, j and k, as:

$$\rho_{jk} = EDL_j T / \sqrt{\left(e^{\sigma_j^2 T} - 1\right)\left(e^{\sigma_k^2 T} - 1\right)}$$

where:

j is the class or classes of the firm's debt or similar securities issued by, or referenced to, the firm for which the expected default loss is being estimated

k is the class or classes of security issued by, or referenced to, the firm that rank behind security j in terms of priority upon a liquidation or default event

- T is the time horizon of interest to the user, in years
- σ_i is the standard deviation of rates of return, per annum, of j
- σ_k is the standard deviation of rates of return, per annum, of k

 EDL_i the annualised expected default loss on security j;

fit the model; and

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output parameters of interest from the fitted model to a user.

10 66. The computer system of claim 64, wherein the processing unit is further operative to:

designate the correlation (ρ_{ik}) of the returns for two said securities j and k, as:

$$\rho_{jk} = EDL_j / \sigma_j \sigma_k ;$$

fit the model; and

- output parameters of interest from the fitted model to the user.
 - 67. The computer system of claim 52, wherein one or more of the securities being analysed by the said system is an option;

the financial modelling unit is further operative to specify the real world distribution process that the returns on the underlying asset are expected to follow;

the financial modelling unit is further operative to calculate the expected real world probability of the option being exercised;

the risk pricing unit is further operative to calculate the expected mean of the option, at the time the option is expected to be exercised;

the risk analysis unit is further operative to calculate the standard deviation and other higher moments of interest of the option, at the time the option is expected to be exercised;

the financial modelling unit is further operative to use the aforesaid parameters to calculate the expected real world pay off from the option;

the risk pricing unit is further operative to discount back to present value (as at

the chosen evaluation date) the pay off from the option using a risk adjusted discount rate, where said risk adjusted discount rate includes a risk premium for the expected standard deviation (measured over discrete time) of the expected option pay off, for such other higher moments of interest to the user and adjustments for any other factors specified by a user, such that the price per unit of risk, for each risk factor, is equated for all of the options being evaluated, for the underlying asset and for any other securities of interest referenced thereto; and

the user interface is further operative to provide as output to a user parameters of interest from the fitted model.

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68. The computer system of claim 52, wherein one or more of the securities being analysed by the said system is an option;

the financial modelling unit is further operative to specify the real world distribution process that the returns on the underlying asset, upon which the option is contingent, are expected to follow, using initial guesses for the values of the distribution parameters;

the financial modelling unit is further operative to calculate the expected real world probability of the option being exercised;

the risk pricing unit is further operative to calculate the expected mean of the option, at the time the option is expected to be exercised;

the risk analysis unit is further operative to calculate the standard deviation and other higher moments of interest of the option, at the time the option is expected to be exercised;

the financial modelling unit is further operative to use the aforesaid parameters to calculate the expected real world pay off from each option;

the risk pricing unit is further operative to discount back to present value (as at the chosen evaluation date) the expected pay offs from each option using a risk adjusted discount rate specific to each option, where said risk adjusted discount rate includes a risk premium for the expected standard deviation of the expected option pay off, for such other higher moments of interest to the user and adjustments for any other factors specified by a user, such that the price per unit of risk, for each risk factor, is equated for all of the options being evaluated, for the underlying asset and for any other

securities of interest referenced thereto;

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the financial modelling unit is further operative to iterate the above process, by varying the model parameter values, until the model is fitted; and

the user interface is further operative to provide as output to a user parameters of interest from the fitted model.

- 69. The computer system of claim 67 or 68, wherein the financial modelling, risk analysis and risk pricing units are further operative to use the estimated values for the rate of return, standard deviation, other higher moments of interest and any other factors specified by a user for the asset, derived as output from said claims, as input to price or value other options contingent on the same or similar assets.
- 70. The computer system of claim 52, wherein the user applies an option-theoretic model of the firm;

the financial modelling unit is further operative to determine a plurality of input parameters, the parameters including a risk premium in the rate of return for each security issued by, or referenced to, the firm;

the financial modelling unit is further operative to define relationships between said parameters;

the financial modelling unit is further operative to fit the model; and the user interface is further operative to provide as output to a user parameters of interest from the fitted model.

71. The computer system of claim 70, wherein:

the financial modelling unit is further operative to specify the real world distribution process that the returns on the firm's assets are expected to follow;

the financial modelling unit is further operative to specify a default point representing the value of the firm's assets at which the firm is expected to default;

the financial modelling unit is further operative to calculate the expected real world probability of the default point being met;

the risk pricing unit is further operative to calculate the expected mean of the securities being analysed, having regard to the distribution process modelled for the

firm's assets and the default point, at the time horizon of interest;

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the risk analysis unit is further operative to calculate the standard deviation and other higher moments of interest of the securities being analysed, having regard to the distribution process modelled for the firm's assets and the default point, at the time horizon of interest;

the financial modelling unit is further operative to use the aforesaid parameters to calculate the expected real world pay off of the securities being analysed, at the time horizon of interest;

the risk pricing unit is further operative to discount back to present value (as at the chosen evaluation date) the expected pay offs of each security being analysed using a risk adjusted discount rate, where said risk adjusted discount rate includes a risk premium for the expected standard deviation of the expected pay off from the security, for such other higher moments of interest to the user and adjustments for any other factors specified by a user, such that the price per unit of risk, for each such risk factor, is equated for two or more securities issued by, or referenced to, the firm;

the financial modelling unit is further operative to fit the model; and the user interface is further operative to provide as output to a user parameters of interest from the fitted model.

72. The computer system of claims 70 or claim-71, wherein the financial modelling unit is further operative to:

define additional multi-variate equations representing relationships between some or all of the variables used in the financial modelling unit in the said claims; and

solve all of the multi-variate equations and the model used in the said claims, to calculate the remaining unknown variables in the equations and the model.

73. The computer system of claim 72, wherein the financial modelling unit is further operative to include as at least one of the unknown inputs in one or more additional multi-variate equations a statistical moment of one of the securities issued by, or referenced to the firm.

74. The computer system of claim 72, wherein the financial modelling unit is further operative to include as at least one of the unknown inputs in one or more additional multi-variate equations the correlation between the returns of a pair of securities issued by, or referenced to, the firm.

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75. The computer system of claim 72, wherein the financial modelling unit is further operative to include as at least one of the unknown inputs in one or more additional multi-variate equations the covariance between the returns of a pair of securities issued by, or referenced to, the firm.

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76. The computer system of claim 72, wherein the financial modelling unit is further operative to include as at least one of the unknown inputs in one or more additional multi-variate equations the correlation between the returns of a security issued by, or referenced to, the firm and the returns of the total firm.

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77. The computer system of claim 72, wherein the financial modelling unit is further operative to include as at least one of the unknown inputs in one or more additional multi-variate equations the covariance between the returns of a security issued by, or referenced to, the firm and the returns of the total firm.

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78. The computer system of claim 72, wherein the financial modelling unit is further operative to include as at least one of the unknown inputs in one or more additional multi-variate equations the expected probability of default.

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79. The computer system of claim 72, wherein the financial modelling unit is further operative to include as at least one of the unknown inputs in one or more additional multi-variate equations the expected loss given default on a debt-type security issued by, or referenced to, the firm.

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80. The computer system of claim 72, wherein the financial modelling unit is further operative to include as at least one of the unknown inputs in one or more additional

multi-variate equations the expected default loss on a debt-type security issued by, or referenced to, the firm.

- 81. The computer system of claim 70 or claim 71, wherein the financial modelling unit is further operative to generate one or more parameters from the model and solve the model so that the said parameters equal values specified by a user, where one of the said parameters is a statistical moment of the returns of one of the securities issued by, or referenced to, the firm.
- 10 82. The computer system of claim 70 or claim 71, wherein the financial modelling unit is further operative to generate one or more parameters from the model and solve the model so that the said parameters equal values specified by a user, where one of the said parameters is the correlation between the returns of a pair of securities issued by, or referenced to, the firm.

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- 83. The computer system of claim 70 or claim 71, wherein the financial modelling unit is further operative to generate one or more parameters from the model and solve the model so that the said parameters equal values specified by a user, where one of the said parameters is the covariance between the returns of a pair of securities issued by, or referenced to, the firm.
- 84. The computer system of claim 70 or claim 71, wherein the financial modelling unit is further operative to generate one or more parameters from the model and solve the model so that the said parameters equal values specified by a user, where one of the said parameters is the correlation between the returns of a security issued by, or referenced to, the firm and the returns of the total firm.
- 85. The computer system of claim 70 or claim 71, wherein the financial modelling unit is further operative to generate one or more parameters from the model and solve the model so that the said parameters equal values specified by a user, where one of the said parameters is the covariance between the returns of a security issued by, or referenced to, the firm and the returns of the total firm.

86. The computer system of claim 70 or claim 71, wherein the financial modelling unit is further operative to generate one or more parameters from the model and solve the model so that the said parameters equal values specified by a user, where one of the said parameters is the expected probability of default.

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- 87. The computer system of claim 70 or claim 71, wherein the financial modelling unit is further operative to generate one or more parameters from the model and solve the model so that the said parameters equal values specified by a user, where one of the said parameters is the expected loss given default on a debt-type security issued by, or referenced to, the firm.
- 88. The computer system of claim 70 or claim 71, wherein the financial modelling unit is further operative to generate one or more parameters from the model and solve the model so that the said parameters equal values specified by a user, where one of the said parameters is the expected default loss on a debt-type security issued by, or referenced to, the firm.
- 89. The computer system of any one of claims 53, 56 or 57 or any one of claims 70 to 88, wherein the financial modelling unit is further operative to derive the value for, or the additional multi-variate equation for solving for, the default loss on a security from applying the computer system of any one of claims 58, 59, 61 or 62.
 - 90. The computer system of any one of claims 53, 56 or 57 or any one of claims 70 to 88, wherein the financial modelling unit is further operative to derive the value for, or the additional multi-variate equation for solving for, the correlation between a pair of securities from applying the computer system of any one of claims 64 to 66.
- 91. The computer system of any one of claims 70 to 88, wherein the financial modelling unit is further operative to derive the values for, or the additional multivariate equations for solving for, the rate of return, standard deviation, other higher

moments of interest and any other relevant factors specified by a user for one or more of the securities from the computer system of claim 67 or claim 68.

- 92. The computer system of any one of claims 67, 68 or 71, wherein the financial modelling unit is further operative to model the real world distribution process that the returns on the firm (or underlying asset) are expected to follow as a specified statistical distribution, wherein the mean, standard deviation and other higher moments of interest of the portions of that distribution relevant to a security are estimated using closed-form type formula solutions or numerical approximations appropriate for the specified statistical distribution process.
 - 93. The computer system of claim 92, wherein the financial modelling unit is further operative to model the real world statistical distribution process that the returns on the firm (or underlying asset) are expected to follow as the normal distribution.

94. The computer system of claim 93, wherein the firm has, or is treated as having, only a single class of zero coupon debt on issue and the financial modelling unit is further operative to fit the model such that:

$$\frac{\ln\left(\frac{V_{0}e^{r_{v}T}\left[1-N(d_{1})\right]+B_{0}e^{r_{v}T}N(d_{2})}{B_{0}}\right)/T-r}{\sigma_{B}} = \frac{\ln\left(\frac{V_{0}e^{r_{v}T}N(d_{1})-B_{0}e^{r_{v}T}N(d_{2})}{S_{0}}\right)/T-r}{\sigma_{S}}$$

where:

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 S_n is the value of the equity of the firm at time n

 V_n is the value of the firm's assets at time n and the value of the firm's assets is the sum of the values of the firm's debt (B) and equity (S)

X is the face value of the firm's debt (B), which is assumed to be a single zerocoupon bond, at maturity

T is the user selected time horizon, in years

 r_{ν} is the rate of return on the firm's assets, per annum

y is the promised yield on the firm's debt, per annum

$$d_{1} = \left(\left[\ln \left(\sqrt[V_{0}]{X} \right) + r_{v} T \right] / \sigma_{v} \sqrt{T} \right) + \left(1/2 \right) \left(\sigma_{v} \sqrt{T} \right)$$

$$d_2 = d_1 - \sigma_{\nu} \sqrt{T}$$

- N(.) is the cumulative probability of the standard normal distribution with d_1 or d_2 as the upper limit
- r is the risk free rate of return, per annum
- σ_{ν} is the standard deviation of rates of return on the firm's assets, per annum
- σ_R is the standard deviation of rates of return on the firm's debt, per annum
- $\sigma_{\scriptscriptstyle S}$ is the standard deviation of rates of return on the firm's equity, per annum.
- 95. A system for applying an option-theoretic model of a firm, the system 10 comprising:
 - a computer-readable memory;
 - a processing unit operative to generate one or more parameters from the said option-theoretic model, estimated over a discrete time period, and solve the model so that the said parameters equal values specified by a user.

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- 96. The computer system of claim 95, wherein the processing unit is further operative to use as one of the said parameters a statistical moment of the returns of one or more of the securities issued by, or referenced to, the firm.
- 20 97. The computer system of claim 95, wherein the processing unit is further operative to use as one of the said parameters the correlation between the returns of a pair of securities issued by, or referenced to, the firm.
- 98. The computer system of claim 95, wherein the processing unit is further operative to use as one of the said parameters the covariance between the returns of a pair of securities issued by, or referenced to, the firm.
 - 99. The computer system of claim 95, wherein the processing unit is further operative to use as one of the said parameters the correlation between the returns of a security issued by, or referenced to, the firm and the returns of the total firm.

100. The computer system of claim 95, wherein the processing unit is further operative to use as one of the said parameters the covariance between the returns of a security issued by, or referenced to, the firm and the returns of the total firm.

5 101. The computer system of any one of claims 70 to 73, claim 81 or claims 92 to 100, wherein the processing unit is further operative to use formula for calculating additional parameters, being instantaneous volatility, for calibration with the model, said formula comprising:

$$\sigma_{B} = \sigma_{V} \frac{V_{0}}{B_{0}} e^{(r_{V} - r_{B})T} [1 - N(d_{1})]$$

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$$\sigma_S = \sigma_V \frac{V_0}{S_0} e^{(r_V - r_S)T} N(d_1)$$

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102. The computer system of any one of claims 70 to 73, claims 81 to 85, or claims 92 to 100, wherein the processing unit is further operative to use formula for calculating additional parameters, being discrete time volatility, correlation and covariance, for calibration with the model, said formula comprising:

$$\sigma_{B} = \sqrt{\ln \left(\frac{V_{T}^{2} \left[1 - N(d_{3}) \right] e^{\sigma_{V}^{2} T} + X^{2} N(d_{2})}{B_{T}^{2}} \right) / T}$$

$$\sigma_{S} = \sqrt{\ln\left(\frac{V_{T}^{2}N(d_{3})e^{\sigma_{V}^{2}T} - 2V_{T}XN(d_{1}) + X^{2}N(d_{2})}{S_{T}^{2}}\right)} / T$$

$$\rho_{BS} = \frac{X - B_{T}}{B_{T}\sqrt{\left(e^{\sigma_{S}^{2}T} - 1\right)\left(e^{\sigma_{B}^{2}T} - 1\right)}}$$

$$\rho_{VB} = \frac{V_{T}\left[1 - N(d_{3})\right]e^{\sigma_{V}^{2}T} + XN(d_{1}) - B_{T}}{B_{T}\sqrt{\left(e^{\sigma_{V}^{2}T} - 1\right)\left(e^{\sigma_{B}^{2}T} - 1\right)}}$$

$$\rho_{VS} = \frac{V_{T}N(d_{3})e^{\sigma_{V}^{2}T} - XN(d_{1}) - S_{T}}{S_{T}\sqrt{\left(e^{\sigma_{V}^{2}T} - 1\right)\left(e^{\sigma_{S}^{2}T} - 1\right)}}$$

$$\sigma_{BS} = \rho_{BS}\sigma_{B}\sigma_{S}$$

$$\sigma_{VB} = \rho_{VB}\sigma_{V}\sigma_{B}$$

$$\sigma_{VS} = \rho_{VS}\sigma_{V}\sigma_{S}$$

where the additional term is:

$$10 d_3 = d_1 + \sigma_V \sqrt{T}$$

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103. A computer-readable medium having computer-executable instructions for performing a method relating a price or value of a plurality of securities associated with an underlying asset, the rate of return on said securities and the risk attributes of said securities, the method comprising:

determining a risk premium incorporated in the rate of return for each security;

designating that a priced risk factor incorporated in the risk premium for each security is the volatility of returns, measured over discrete time, and that the price per unit of this risk factor is the same for two or more of the said securities; and

defining a model comprising data representing relationships between the risk premiums determined for each security.

104. The computer-readable medium of claim 103, wherein at least one of said plurality of securities is a debt-type instrument, and further comprising computer-executable instructions for analysing a yield spread associated with the debt-type

instrument and identifying a default loss component and a risk premium component of said yield spread.

105. The computer-readable medium of claim 103, further comprising computer - executable instructions to fit the model.

- 106. The computer-readable medium of claim 105, further comprising computer-executable instructions to output to a user parameters of the fitted model.
- 10 107. The computer-readable medium of claim 103, further comprising computer-executable instructions for utilising an estimate of the expected default loss of another, debt-type security (security *j*) issued by, or referenced to, the firm, in analysing the rate of return for a security (or securities) issued by, or referenced to, a firm, said analysis comprising:

determining the rate of return on security $j(r_j)$ by reference to the promised yield on said security (y_j) and the expected default loss (EDL_j) on said security where:

$$r_i = y_i - EDL_i$$

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calculating the excess return for said security j as equal to $r_j - r$, where r is the risk free rate of return;

calculating the exposure of each security to each priced risk factor (m);

calculating a price per unit of risk (λ_m) for each priced risk factor (m) in which each λ_m is the same for two or more securities issued by, or referenced to, the firm and such that the product of the risk exposures for security j and the prices per unit of risk equals the excess return for security j, and similarly for any other security for which an estimate of the excess return is available;

designating that one of the priced risk factors relates to the volatility of the rate of return on securities estimated over a discrete time period and is specific to securities issued by, or referenced to, the firm;

calculating the excess rate of return for all of the other securities being analysed, other than j, based at least partly on their exposure to each priced risk factor and the price per unit of risk for each risk factor;

fitting the model; and providing as output to a user parameters of interest from the fitted model.

108. The computer-readable medium of claim 107, wherein the only risk factor priced in accordance with the computer-executable instructions is the volatility of returns and the said computer-executable instructions:

designate the relationship between the firm specific price of volatility risk (λ_{σ}) , the rate of return for $j(r_j)$, the volatility of returns for $j(\sigma_j)$ and the risk free rate of return (r) as:

$$\lambda_{\sigma} = \frac{r_{j} - r}{\sigma_{j}}$$

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designate the rate of return (r_k) on another class, or classes, of security (k) issued by, or referenced to, the firm as:

$$r_k = r + \lambda_{\sigma} \sigma_k$$

designate, where security class or classes k are debt-type securities, the default loss on said securities by combining the promised yield on said securities (y_k) and their rate of return (r_k) as follows:

$$EDL_k = y_k - r_k$$

fit the model; and

provide as output to a user parameters of interest from the fitted model.

109. A computer readable medium having computer-executable instructions for estimating the covariance of returns for two assets, wherein said covariance is used as a measure of credit risk of one of the assets.

25 110. The computer-readable medium of claim 109, wherein the two assets analysed in accordance with the computer-executable instructions are securities issued by, or referenced to, the same firm, and said covariance output is used as a measure of credit risk of the security that ranks highest in priority upon a liquidation or default event.

111. A computer readable medium having computer-executable instructions for estimating the expected default loss of a security, wherein said estimate of expected default loss is used as a measure of the covariance of returns for that security and another security issued by, or referenced to, the same firm wherein the first security ranks higher in priority upon a liquidation or default event.

112. The computer-readable medium of any one of claims 109 to 111, wherein the computer-executable instructions:

designate the annualised expected default loss (EDL_j) on one of the said securities, security j, as:

$$EDL_{j} = \ln \left(\rho_{jk} \sqrt{\left(e^{\sigma_{j}^{2}T} - 1\right)\left(e^{\sigma_{k}^{2}T} - 1\right)} + 1 \right) / T$$

where:

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j is the class or classes of the firm's debt-type or similar securities issued by, or referenced to, the firm for which the expected default loss is being estimated

k is the class or classes of security issued by, or referenced to, the firm that rank behind security j in terms of priority upon a liquidation or default event

T is the time horizon of interest to the user, in years

 σ_j is the standard deviation of rates of return, per annum, of j

 σ_k is the standard deviation of rates of return, per annum, of k

 ρ_{jk} is the correlation coefficient of the rates of return for j and k;

fit the model; and

output parameters of interest from the fitted model to a user.

113. The computer-readable medium of any one of claims 109 to 111, wherein the computer-executable instructions:

designate the annualised expected default loss (EDL_j) on one of the said

securities, security j, as:

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$$EDL_i = \rho_{ik}\sigma_i\sigma_k$$

fit the model; and

provide as output to a user parameters of interest from the fitted model.

- 114. The computer-readable medium of claim 107, wherein the two assets analysed with the computer-executable instructions are portfolios or indices in respect of different types of security and said covariance output is used as a measure of credit risk.
- 115. A computer-readable medium having computer-executable instructions for estimating the correlation of returns for two securities issued by, or referenced to, a firm by relating the said correlation to computer generated estimates of the variance of the said securities and the expected default loss of one of the said securities.

116. The computer-readable medium of claim 115, wherein the computer-executable instructions:

designate the correlation (ρ_{jk}) of the returns for the two said securities, j and k, as:

$$20 \rho_{jk} = EDL_jT / \sqrt{\left(e^{\sigma_j^2T} - 1\right)\left(e^{\sigma_k^2T} - 1\right)}$$

where:

- j is the class or classes of the firm's debt or similar securities issued by, or referenced to, the firm for which the expected default loss is being estimated
- k is the class or classes of security issued by, or referenced to, the firm that rank behind security j in terms of priority upon a liquidation or default event
- T is the time horizon of interest to the user, in years
- σ_i is the standard deviation of rates of return, per annum, of j
- σ_k is the standard deviation of rates of return, per annum, of k

 EDL_i the annualised expected default loss on security j;

fit the model; and

output parameters of interest from the fitted model to a user.

117. The computer-readable medium of claim 115, wherein the computer-executable instructions:

designate the correlation (ρ_{ik}) of the returns for two said securities j and k, as:

$$\rho_{jk} = EDL_j / \sigma_j \sigma_k$$
;

fit the model; and

output parameters of interest from the fitted model to the user.

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118. The computer-readable medium of claim 103, wherein one or more of the securities being analysed with the computer-executable instructions is an option, and the said computer-executable instructions:

specify the real world distribution process that the returns on the underlying asset are expected to follow;

calculate the expected real world probability of the option being exercised;

calculate the expected mean, standard deviation and other higher moments of interest of the option, at the time the option is expected to be exercised;

use the aforesaid parameters to calculate the expected real world pay off from the option;

discount back to present value (as at the chosen evaluation date) the pay off from the option using a risk adjusted discount rate, where said risk adjusted discount rate includes a risk premium for the expected standard deviation (measured over discrete time) of the expected option pay off, for such other higher moments of interest to the user and adjustments for any other factors specified by a user, such that the price per unit of risk, for each risk factor, is equated for all of the options being evaluated, for the underlying asset and for any other securities of interest referenced thereto; and

provide as output to a user parameters of interest from the fitted model.

30 119. The computer-readable medium of claim 103, wherein one or more of the securities being analysed with the computer-executable instructions is an option and the computer-executable instructions:

specify the real world distribution process that the returns on the underlying asset, upon which the option is contingent, are expected to follow, using initial guesses for the values of the distribution parameters;

calculate the expected real world probability of the option being exercised;

calculate the expected mean, standard deviation and other higher moments of interest of the option, at the time the option is expected to be exercised;

use the aforesaid parameters to calculate the expected real world pay off from each option;

discount back to present value (as at the chosen evaluation date) the expected pay offs from each option using a risk adjusted discount rate specific to each option, where said risk adjusted discount rate includes a risk premium for the expected standard deviation of the expected option pay off, for such other higher moments of interest to the user and adjustments for any other factors specified by a user, such that the price per unit of risk, for each risk factor, is equated for all of the options being evaluated, for the underlying asset and for any other securities of interest referenced thereto;

iterate the above process, by varying the model parameter values, until the model is fitted; and

provide as output to a user parameters of interest from the fitted model.

20 120. The computer-readable medium of claim 118 or claim 119, wherein the computer-executable instructions use the estimated values for the rate of return, standard deviation, other higher moments of interest and any other factors specified by a user for the asset, derived as output from said claims, as input to price or value other options contingent on the same or similar assets.

121. The computer-readable medium of claim 103, wherein a user applies an option-theoretic model of the firm and the computer-executable instructions:

determine a plurality of input parameters, the parameters including a risk premium in the rate of return for each security issued by, or referenced to, the firm;

define relationships between said parameters;

fit the model; and

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provide as output to a user parameters of interest from the fitted model.

122. The computer-readable medium of claim 121, wherein the computer-executable instructions:

specify the real world distribution process that the returns on the firm's assets are expected to follow;

specify a default point representing the value of the firm's assets at which the firm is expected to default;

calculate the expected real world probability of the default point being met;

calculate the expected mean, standard deviation and other higher moments of interest of the securities being analysed, having regard to the distribution process modelled for the firm's assets and the default point, at the time horizon of interest;

use the aforesaid parameters to calculate the expected real world pay off of the securities being analysed, at the time horizon of interest;

discount back to present value (as at the chosen evaluation date) the expected pay offs of each security being analysed using a risk adjusted discount rate, where said risk adjusted discount rate includes a risk premium for the expected standard deviation of the expected pay off from the security, for such other higher moments of interest to the user and adjustments for any other factors specified by a user, such that the price per unit of risk, for each such risk factor, is equated for two or more securities issued by, or referenced to, the firm;

fit the model; and

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provide as output to a user parameters of interest from the fitted model.

123. The computer-readable medium of claim 121 or 122, wherein the computer-executable instructions:

define additional multi-variate equations representing relationships between some or all of the variables used in the models in claims 121 or 122; and

solve all of the multi-variate equations and the said model to calculate the remaining unknown variables in the equations and the model.

124. The computer-readable medium of claim 123, wherein the computer-executable instructions include as at least one of the unknown inputs in one or more additional

multi-variate equations a statistical moment of one of the securities issued by, or referenced to the firm.

- 125. The computer-readable medium in claim 123, wherein the computer-executable instructions include as at least one of the unknown inputs in one or more additional multi-variate equations the correlation between the returns of a pair of securities issued by, or referenced to, the firm.
- 126. The computer-readable medium of claim 123, wherein the computer-executable instructions include as at least one of the unknown inputs in one or more additional multi-variate equations the covariance between the returns of a pair of securities issued by, or referenced to, the firm.
 - 127. The computer-readable medium of claim 123, wherein the computer-executable instructions include as at least one of the unknown inputs in one or more additional multi-variate equations the correlation between the returns of a security issued by, or referenced to, the firm and the returns of the total firm.

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- 128. The computer-readable medium of claim 123, wherein the computer-executable instructions include as at least one of the unknown inputs in one or more additional multi-variate equations the covariance between the returns of a security issued by, or referenced to, the firm and the returns of the total firm.
- 129. The computer-readable medium of claim 123, wherein the computer-executable instructions include as at least one of the unknown inputs in one or more additional multi-variate equations the expected probability of default.
 - 130. The computer-readable medium of claim 123, wherein the computer-executable instructions include as at least one of the unknown inputs in one or more additional multi-variate equations the expected loss given default on a debt-type security issued by, or referenced to, the firm.

131. The computer-readable medium of claim 123, wherein the computer-executable instructions include as at least one of the unknown inputs in one or more additional multi-variate equations the expected default loss on a debt-type security issued by, or referenced to, the firm.

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- 132. The computer-readable medium of claim 121 or claim 122, wherein the computer-executable instructions generate one or more parameters from the model and solve the model so that the said parameters equal values specified by a user, where one of the said parameters is a statistical moment of the returns of one of the securities issued by, or referenced to, the firm.
- 133. The computer-readable medium of claim 121 or claim 122, wherein the computer-executable instructions generate one or more parameters from the model and solve the model so that the said parameters equal values specified by a user, where one of the said parameters is the correlation between the returns of a pair of securities issued by, or referenced to, the firm.
- 134. The computer-readable medium of claim 121 or claim 12, wherein the computer-executable instructions generate one or more parameters from the model and solve the model so that the said parameters equal values specified by a user, where one of the said parameters is the covariance between the returns of a pair of securities issued by, or referenced to, the firm.
- 135. The computer-readable medium of claim 121 or claim 122, wherein the computer-executable instructions generate one or more parameters from the model and solve the model so that the said parameters equal values specified by a user, where one of the said parameters is the correlation between the returns of a security issued by, or referenced to, the firm and the returns of the total firm.
- 30 136. The computer-readable medium of claim 121 or claim 122, wherein the computer-executable instructions generate one or more parameters from the model and solve the model so that the said parameters equal values specified by a user, where one

of the said parameters is the covariance between the returns of a security issued by, or referenced to, the firm and the returns of the total firm.

- The computer-readable medium of claim 121 or claim 122, wherein the 137. computer-executable instructions generate one or more parameters from the model and solve the model so that the said parameters equal values specified by a user, where one of the said parameters is the expected probability of default.
- 138. The computer-readable medium of claim 121 or claim 122, wherein the computer-executable instructions generate one or more parameters from the model and 10 solve the model so that the said parameters equal values specified by a user, where one of the said parameters is the expected loss given default on a debt-type security issued by, or referenced to, the firm.
- 15 The computer-readable medium of claim 121 or claim 122, wherein the 139. computer-executable instructions generate one or more parameters from the model and solve the model so that the said parameters equal values specified by a user, where one of the said parameters is the expected default loss on a debt-type security issued by, or referenced to, the firm.

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- The computer-readable medium of any one of claims 104, 107 or 108 or any one 140. of claims 121 to 139, wherein the computer-executable instructions derive the value for, or the additional multi-variate equation for solving for, the default loss on a security from applying the computer-executable instructions contained in the computer-readable medium of any one of claims 109, 110, 112 or 113.
- The computer-readable medium of any one of claims 104, 107 or 108 or any one of claims 121 to 139, wherein the computer-executable instructions derive the value for, or the additional multi-variate equation for solving for, the correlation between a pair of securities is derived from applying the computer-executable instructions contained in 30

the computer-readable medium of any one of claims 115 to 117.

142. The computer-readable medium of any one of claims 121 to 139, wherein the computer-executable instructions derive the values for, or the additional multi-variate equations for solving for, the rate of return, standard deviation, other higher moments of interest and any other relevant factors specified by a user for one or more of the securities from applying the computer-executable instructions contained in the computer-readable medium of claim 118 or claim 119.

143. The computer-readable medium of any one of claims 118, 119 or 122, wherein computer-executable instructions model the real world distribution process that the returns on the firm (or underlying asset) are expected to follow as a specified statistical distribution, wherein the mean, standard deviation and other higher moments of interest of the portions of that distribution relevant to a security are estimated using closed-form type formula solutions or numerical approximations appropriate for the specified statistical distribution process.

144. The computer-readable medium of claim 143, wherein the computer-executable instructions model the real world statistical distribution process that the returns on the firm (or underlying asset) are expected to follow as the normal distribution.

20 145. The computer-readable medium of claim 144, wherein the firm has, or is treated as having, only a single class of zero coupon debt on issue and the computer-executable instructions fit the model such that:

$$\frac{\ln\!\left(\frac{V_0 e^{\eta_r T} \left[\!\left[\!1 - N\!\left(\!d_1\right)\!\right]\! + B_0 e^{\eta_r T} N\!\left(\!d_2\right)\!\right]\!\right)\!\!\left/\!T - r}{\sigma_B} = \frac{\ln\!\left(\frac{V_0 e^{\eta_r T} N\!\left(\!d_1\right)\! - B_0 e^{\eta_r T} N\!\left(\!d_2\right)\!\right)\!\!\left/\!T - r}{S_0}\right)\!\!\left/\!T - r\right.}{\sigma_S}$$

25 where:

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- S_n is the value of the equity of the firm at time n
- V_n is the value of the firm's assets at time n and the value of the firm's assets is the sum of the values of the firm's debt (B) and equity (S)
- X is the face value of the firm's debt (B), which is assumed to be a single zerocoupon bond, at maturity
- T is the user selected time horizon, in years

 r_{ν} is the rate of return on the firm's assets, per annum

y is the promised yield on the firm's debt, per annum

$$d_{1} = \left(\left[\ln \left(\frac{V_{0}}{X} \right) + r_{v}T \right] / \sigma_{v} \sqrt{T} \right) + (1/2) \left(\sigma_{v} \sqrt{T} \right)$$

$$d_2 = d_1 - \sigma_V \sqrt{T}$$

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5 N(.) is the cumulative probability of the standard normal distribution with d_1 or d_2 as the upper limit

r is the risk free rate of return, per annum

 σ_V is the standard deviation of rates of return on the firm's assets, per annum

 $\sigma_{\scriptscriptstyle B}$ is the standard deviation of rates of return on the firm's debt, per annum

 σ_s is the standard deviation of rates of return on the firm's equity, per annum.

146. A computer readable medium having computer-executable instructions for performing a method to apply an option-theoretic model of a firm, said method comprising the steps of generating one or more parameters from the model, estimated over a discrete time period, and solving the model so that the said parameters equal values specified by a user.

- 147. The computer-readable medium of claim 146, wherein one of the said parameters analysed by the computer-executable instructions is a statistical moment of the returns of one or more of the securities issued by, or referenced to, the firm.
- 148. The computer-readable medium of claim 146, wherein one of the said parameters analysed by the computer-executable instructions is the correlation between the returns of a pair of securities issued by, or referenced to, the firm.

149. The computer-readable medium of claim 146, wherein one of the said parameters analysed by the computer-executable instructions is the covariance between the returns of a pair of securities issued by, or referenced to, the firm.

30 150. The computer-readable medium of claim 146, wherein one of the said parameters analysed by the computer-executable instructions is the correlation between

the returns of a security issued by, or referenced to, the firm and the returns of the total firm.

- 151. The computer-readable medium of claim 146, wherein one of the said parameters analysed by the computer-executable instructions is the covariance between the returns of a security issued by, or referenced to, the firm and the returns of the total firm.
- 152. The computer-readable medium of any one of claims 121 to 124, claim 132 or claims 143 to 151, wherein the computer-executable instructions use formula for calculating additional parameters, being instantaneous volatility, for calibration with the model, said formula comprising:

$$\sigma_{B} = \sigma_{V} \frac{V_{0}}{B_{0}} e^{(r_{V} - r_{B})T} \left[1 - N(d_{1}) \right]$$

$$\sigma_{S} = \sigma_{V} \frac{V_{0}}{S_{0}} e^{(r_{V} - r_{S})T} N(d_{1})$$

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153. The computer-readable medium of any one of claims 121 to 124, claims 132 to 136, or claims 143 to 151, wherein the computer-executable instructions use formula for calculating additional parameters, being discrete time volatility, correlation and covariance, for calibration with the model, said formula comprising:

$$\sigma_{B} = \sqrt{\ln\left(\frac{V_{T}^{2} \left[1 - N(d_{3})\right] e^{\sigma_{V}^{2}T} + X^{2} N(d_{2})}{B_{T}^{2}}\right) / T}$$

$$\sigma_{S} = \sqrt{\ln\left(\frac{V_{T}^{2}N(d_{3})e^{\sigma_{V}^{2}T} - 2V_{T}XN(d_{1}) + X^{2}N(d_{2})}{S_{T}^{2}}\right)} / T$$

$$\rho_{BS} = \frac{X - B_{T}}{B_{T}\sqrt{\left(e^{\sigma_{S}^{2}T} - 1\right)\left(e^{\sigma_{B}^{2}T} - 1\right)}}$$

$$\rho_{VB} = \frac{V_{T}\left[1 - N(d_{3})\right]e^{\sigma_{V}^{2}T} + XN(d_{1}) - B_{T}}{B_{T}\sqrt{\left(e^{\sigma_{V}^{2}T} - 1\right)\left(e^{\sigma_{B}^{2}T} - 1\right)}}$$

$$\rho_{VS} = \frac{V_{T}N(d_{3})e^{\sigma_{V}^{2}T} - XN(d_{1}) - S_{T}}{S_{T}\sqrt{\left(e^{\sigma_{V}^{2}T} - 1\right)\left(e^{\sigma_{S}^{2}T} - 1\right)}}$$

$$\sigma_{BS} = \rho_{BS}\sigma_{B}\sigma_{S}$$

$$\sigma_{VB} = \rho_{VB}\sigma_{V}\sigma_{B}$$

$$\sigma_{VS} = \rho_{VS}\sigma_{V}\sigma_{S}$$

where the additional term is:

$$10 d_3 = d_1 + \sigma_V \sqrt{T}$$

- 154. A computer-readable medium having stored thereon the output from the process of any one of claims 1 to 51.
- 15 155. A computer-readable medium having stored thereon the output from operating the system of any one of claims 52 to 102.
 - 156. A computer-readable medium having stored thereon the output from executing the computer-executable instructions of any one of claims 103 to 153.
 - 157. A computer-readable medium having stored thereon an order to buy or sell securities, or otherwise enter into a financial contract, based at least in part on output from the process of any one of claims 1 to 51.

158. A computer-readable medium having stored thereon an order to buy or sell securities, or otherwise enter into a financial contract, based at least in part on the output from operating the system of any one of claims 52 to 102.

- 5 159. A computer-readable medium having stored thereon an order to buy or sell securities, or otherwise enter into a financial contract, based at least in part on the output from executing the computer-executable instructions of any one of claims 103 to 153.
- 160. A computer-readable medium having stored thereon a recommendation to buy or sell securities, or otherwise enter into a financial contract, based at least in part on output from the process of any one of claims 1 to 51.
 - 161. A computer-readable medium having stored thereon a recommendation to buy or sell securities, or otherwise enter into a financial contract, based at least in part on the output from operating the system of any one of claims 52 to 102.

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- 162. A computer-readable medium having stored thereon a recommendation to buy or sell securities, or otherwise enter into a financial contract, based at least in part on the output from executing the computer-executable instructions of any one of claims 103 to 153.
- 163. The computer-readable medium of any one of claims 103 to 162, wherein the computer-readable medium includes one or more non-volatile storage devices.
- 25 164. The computer-readable medium of any one of claims 103 to 162, wherein the computer-readable medium includes a propagated data signal.